Tuning and Temperament and why we have to do it

It is axiomatic that no scale on a keyboard instrument can be perfectly in tune. Reaching a tolerable compromise is called tempering. Nature provides a number of exactly in-tune intervals, the harmonic series, of which three of the first sixteen are incompatible with any normal European scale: 7 is very flat, 11 is halfway between F and F♯, 13 is noticeably flat, and 14 is the octave of 7. Natural horn and trumpet players have to pull them into tune with our scales.

The number of each harmonic, starting the series with 1 as the fundamental, also shows their ratio relationship. They double at the octave, for example (2:1, 4:2, 6:3, etc). Touch a string lightly halfway along (2:1), and the octave will sound; touch it a third of the way, the twelfth (3:1) will sound, and so forth. Similarly with the number of vibrations per second (or Hertz). Our tuning A is 440 Hz; the E a pure fifth higher is 660 Hz, the A an octave higher is 880 Hz.

Three reasons that no scale can be in tune are: that twelve fifths piled on top of each other are an eighth of a tone larger than seven octaves similarly piled; that three thirds on top of each other are almost a quarter-tone smaller than an octave; and that two octaves plus one third are almost an eighth of tone smaller than four fifths.

We measure musical intervals with a unit called the cent (our equivalent of a millimetre or cubic centimetre, etc). There are 100 cents in an equal-tempered semitone, and therefore 1200 cents in an octave. Like mm and cc, cents are artificial, but also like them they are always the same size, whereas no two successive tones are the same number of Hertz (Hz) apart.

Building a scale with intervals from the harmonic series, and starting with the 8th harmonic C, the D a tone higher (9:8) is 204 cents above the C, a major tone; the E a tone above (10:9, a smaller interval than 9:8, and thus a minor tone) is 182 cents above the D, and together these add up to the ratio of a pure major third (5:4), 386 cents. The F (4:3) is 498 cents, a pure fourth above the C, and subtracting 386 from 498 gives the distance from E to F, a wide semitone of 112 cents (16:15). The G (3:2) is 702 cents, a pure fifth, above the C. The A and the B have been variously placed by differing authorities, but the most logical solution is to put a pure major third on F which, because F to G is already set at 204 cents (702 minus 498), means that G to A must be 182 cents (386 minus 204). If B to C is to be the same size as E to F, as the ratio 16:15 implies, then A to B must be 204 cents, which permits a pure major third also from G to B. This means that all the major thirds of the diatonic scale are pure:

M	m	½	M	m	M	½		
C	D	E	F	G	A	B	C	M = major tone
204	182	112	204	182	204	112		m = minor tone
0	204	386	498	702	884	1088	1200	½ = semitone
*	386	*	*	386	*			
				*	386	*		

We could set a C major scale on a keyboard instrument, using these harmonic intervals, and this will be perfectly in tune. The problems arise when we wish to change key or use these pitches other than melodically. Because the steps of the major scale are major tone, minor tone, semitone, etc, if we want a D major scale, where the first step should be a major tone we have already set a minor tone from D and E, and there are similar problems all the way up and in all other keys. Thus this tuning, called Just Intonation, cannot be used on a keyboard instrument which, by its nature, must have fixed pitches. It can be used by singers and by instruments whose pitch can be varied slightly, and indeed it is so used by anyone who wants to play or sing perfectly in tune, as it has been since ancient Greek times.

Once keyboard instruments were introduced, a new tuning was necessary. The first, in the Middle Ages, was the Pythagorean, in which all but one of the fifths were pure and all the whole tones but one were major tones of 204 cents. The problem came with the major third. C to E (ratio 5:4) is 386 cents, which is the same as a major tone (204 cents) plus a minor tone (182 cents). When, however, two major tones are added together, the result is a third of 408 cents, an eighth of a tone sharp, 24 cents, called a Pythagorean comma. A chord with tonic, fifth, and a 408-cent third makes a very discordant sound. The third was regarded as a dissonance in the Middle Ages simply because it *was* dissonant. By the mid-15th century, and probably from the late 14th, composers (e.g. in the *Faenza Codex*) took advantage of the fact that four of the eight thirds were almost pure. Arnault de Zwolle, writing about 1440, started his cycle of fifths on B, because that was the lowest note of his clavichord, tuning pure fifth flat-wise from there, B-E, E-A, A-D, D-G, G-C, C-F, F-B♭, B♭-E♭, E♭-A♭, A♭-D♭, D♭-G♭. These twelve fifths total 8424 cents (702 × 12), which is 24 cents, the Pythagorean comma, more than seven octaves (1200 × 7), and therefore his last fifth, from G♭ to B, had to be made small (678 cents: 702 minus 24). As a result, the four thirds D-F♯, A-C♯, E-G♯, and B-D♯, which appear below as diminished fourths, are almost pure (384 cents instead of 386):

B	C	D♭	D♮	E♭	E♮	F	G♭	G♮	A♭	A♮	B♭	B♮
90	90	114	90	114	90	90	114	90	114	90	114	
	*	204	**	204	*	*	204	**	204	**	204	*
-90	0	90	204	294	408	498	588	702	792	906	996	1110
		*	*		384		*			*		384
*		384		*	*		384		*			

When harmony had evolved to the stage where thirds were a necessary part of any chord, a new temperament was needed. The third had to be pure, and the fourths and fifths as pure as possible. This was achieved by taking the average, or mean, size of whole tone, by halving the 386-cent third and so producing a tone of 193 cents, the meantone, a tone between the major and the minor tone. At the same time, the syntonic comma of 22 cents, the difference between four fifths and two octaves plus a third, one of the problems with which we started, was distributed among the fifths, each commonly used fifth being tempered by a quarter of a comma, or 5.5 cents, producing an interval which was slightly out of tune, but not enough to cause too much trouble. This scale is called quarter-comma Meantone, Aron's Meantone (after the first person to describe it fully, in 1523) or just Meantone. This can be constructed by starting on C and tuning the third C-E pure. The fifths within this third, C-G, G-D, D-A, and A-E, are then tempered by tuning each a quarter of a comma (5.5 cents) narrow (696.5). Then pure thirds are tuned both up and down from each of these notes. This gives excellent results save that the pure third below C (A♭) is by no means the same note as the pure third above E (G♯). The A♭ is 814 cents above C, whereas the G♯ is only 772 cents above:

C	C♯	D	E♭	E♮	F	F♯	G	G♯	A	B♭	B♮	C
75.5	117.5	117.5	75.5	117.5	75.5	117.5	75.5	117.5	117.5	75.5	117.5	
*	193	**	193	*	*	193	**	193	**	193	*	
0	75.5	193	310.5	386	503.5	579	696.5	772	889.5	1007	1082.5	1200
							NB:	A♭ = 814				

We have already seen that three thirds are 41 cents less than an octave (386.3 x 3 = 1159 cents); this gap is called a diësis, and this is the size of the so-called wolf fifth which results when A♭ is used instead of G♯ or vice versâ. It gets its name from the fact that a fifth 41 cents out of tune howls like a wolf. The howl is due to the beats between the two notes. When two notes are one or two Hz out of tune, a beat or throb of once or twice a second can be heard. As they get further out of tune, the beat rate increases by the number of Hz they differ. Around the tuning A (440 Hz), 41 cents is 10 Hz, and a beat-rate of ten per second produces quite a howl. The only beat-free intervals are those produced by the whole-number ratios of the harmonic series, which is why Just Intonation is the only scale whose intervals are perfectly in tune. These beats are used when tuning: as one plays the C and the E, one turns the tuning key on the wrest pin of the E, listening to hear the beats slow down until they finally vanish; the third is then pure. Tempering the G above middle C, one first tunes it pure from the C and then flattens it until one can hear one and a quarter beats per second. The other fifths are tempered similarly though the beat rate differs according to the pitch and to which octave one is working in.

There are also four wolf thirds (strictly, diminished fourths again) of 428 cents in Meantone, C♯-F, F♯-B♭, B-E♭, and G♯-C. These are all in keys which come at the extremes of the cycle of fifths, keys which composers took care to avoid. However, there was no necessity to start tuning from C, and by moving the starting point one or more fifths in either direction, the wolves could be shifted. A way that was often employed to avoid the wolf fifth was to split a key so that the front half controlled strings or pipes tuned to G♯, and the back half to A♭, but it was seldom practicable to avoid all the wolves in this way, despite a number of attempts to do so with keyboards with 19, 31, or even more keys to the octave.

When a fairly fully chromatic range was wanted, temperament had to proceed further. Smaller fractions of the comma were tried, 5th comma, 6th comma, and 8th comma. The abiding problem is that as one improves the third, inevitably one worsens the fifth. The system used on pianos today, Equal Temperament, was devised before 1600 (an approximation to it was used on fretted strings such as viols and lutes much earlier). In this temperament everything is out of tune except the octave. It ignores all natural intervals except the octave, and instead it is a logarithmic scale based on the twelfth root of two. The fifth and the fourth are tolerable, and much better than in Meantone, being only two cents out (700 instead of 702, and 500 instead of 498), but the thirds are much worse (14 cents out; 400 instead of 386), which is why even as late as the Great Exhibition of 1851, all the organs are said still to have been tuned in Meantone, though this was not the normal quarter-comma Meantone. It is also why non-keyboard musicians, on the whole, tried to avoid it until Schönberg and his theories made it inevitable.

What was used instead was the various irregular temperaments such as those of Vallotti, Werckmeister, Kirnberger, Young and others. These were based on improving as far as possible the keys which one was most likely to use, and allowing those less often used to be fairly dirty. As already noted, one did not, of course, always have to tune from C; when playing in E♭, one could always start there and let A major suffer. It has been suggested that something like this was the Well-Tempering that Bach used in *The 48 Preludes and Fugues*. John Barnes, for example, analyzed *The 48* in *Early Music* (April 1979) and, by assuming that those which were slow and with good plain chords should be better in tune than Equal Temperament, while those with much running passage work which could disguise the tuning, could be worse, came up with a Bach Well-Temperament. Other scholars, naturally, disagreed and came up with other solutions. I have greatly simplified the argument here.

For those who wish to investigate the subject further, J Murray Barbour *Tunings and Temperaments* (Da Capo Press, 1972; originally Michigan State College, East Lansing, 1951) is a good place to start, as is the article in *The New Grove Dictionary* and *The New Grove Dictionary of Musical Instruments*. There is in the Bate Collection a 12-string monochord on which any temperament can be set quite easily, and a virginals which can be tuned to any temperament desired, but rather more laboriously (always tune down first, to check whether the key is on the right pin). Both may be used by anybody interested in experimenting with temperaments.

For those who wish to calculate cents from Hertz or ratios, using a pocket calculator with logs, the procedure is: [higher ÷ lower] = ... [log(n) x 1731.234 (the constant for log(n); the constant for log(10) is 3986.3137)] = cents. To go the other way, from cents to ratio or Hz: [cents ÷ the constant] = ... [antilog x base Hz or ratio] = Hz or ratio. As a check, always start with something you know the answer to, e.g. [3 ÷ 2] = 1.5 [log(n) (0.4054651) x 1731.234] = 701.95498; call it 702 cents. ALWAYS use the constant for the type of logs being used; the constant for log(n) used with log(10) can only produce a nonsense.

(with thanks for much kind help from Lewis Jones)

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